Name Key

Review of limits and continuity

Calculus Q1 T1

1. Determine the value of c that makes the piecewise-defined function $g(x)$ everywhere continuous. $g(x)=\begin{cases}\sqrt{x+4}, & x<5\\ x^2+c, & x\geq 5\end{cases}$

$\sqrt{5+4}=\sqrt{9}=3$

$5^2+c=3 \rightarrow 25+c=3$ $\boxed{c=-22}$

2. Is $h(x)$ continuous for all real numbers? If so show why.

$h(x)=\begin{cases}x+3, & x\leq -1\\ -x^2, & x>-1\end{cases}$

$-1+3=2$

$-(-1)^2=-1$

No, not continuous when $x=-1$

3. Evaluate $\lim\limits_{h\to 0}\dfrac{(2+h)^2-4}{h}$.

$\dfrac{4+4h+h^2-4}{h} \rightarrow \dfrac{h^2+4h}{h} \rightarrow h+4$

$0+4=$ $\boxed{4}$

4. Evaluate $\lim\limits_{x\to 4}\dfrac{x-4}{\sqrt{x^2-16}}$.

$\cdot\dfrac{\sqrt{x^2-16}}{\sqrt{x^2-16}}=\dfrac{(x-4)(\sqrt{x^2-16})}{x^2-16}=\dfrac{\cancel{(x-4)}(\sqrt{x^2-16})}{\cancel{(x-4)}(x+4)}=\dfrac{\sqrt{x^2-16}}{x+4}=\dfrac{0}{8}$

$\boxed{0}$

5. Evaluate $\lim\limits_{x\to 0}\dfrac{2^x-1}{2^x}$. $=\dfrac{2^0-1}{2^0}=\dfrac{1-1}{1}=\dfrac{0}{1}=\boxed{0}$

6. Evaluate $\lim_{x\to 2}\dfrac{x^2+3x-10}{x-2}$.

$$\frac{(x+5)(x-2)}{(x-2)} = x+5$$

$$2+5 = \boxed{7}$$

7. Evaluate $\lim_{x\to\infty}\dfrac{2x^3-3}{3x^3+25}$.

$$= \boxed{\frac{2}{3}}$$

8. Evaluate $\lim_{x\to\infty}\dfrac{2x^7+3x}{3x^5+2x}$.

$$= \boxed{\infty}$$

9. Evaluate $\lim_{x\to 1}\dfrac{x-1}{\sqrt{x^2+3}\ -\ 2}$.

$$\cdot\frac{\sqrt{x^2+3}\ +2}{\sqrt{x^2+3}\ +2} = \frac{(x-1)(\sqrt{x^2+3}\ +2)}{x^2+3\ -4} = \frac{(x-1)(\sqrt{x^2+3}\ +}{x^2-1}$$

$$\frac{\cancel{(x-1)}(\sqrt{x^2+3}+2)}{\cancel{(x-1)}(x+1)} = \frac{\sqrt{x^2+3}+2}{x+1} = \frac{\sqrt{1^2+3}\ +2}{1+1} = \frac{4}{2} = \boxed{2}$$

10. Evaluate $\lim_{x\to 4}\dfrac{x+14}{\sqrt{x^2-7}}$.

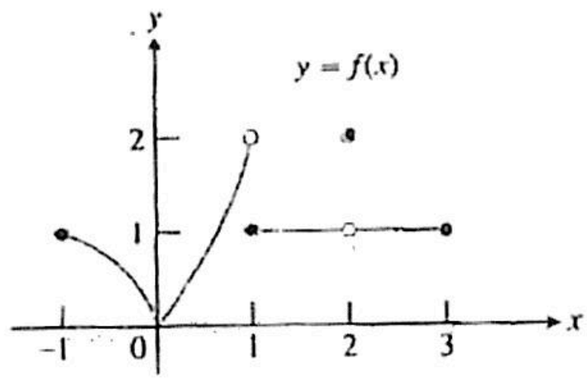
$$\frac{4+14}{\sqrt{4^2-7}} = \frac{18}{\sqrt{9}} = \frac{18}{3} = \boxed{6}$$

In # 11 – 20, which of the statements are true about the function y = f(x) graphed and which are false?

11. Find $\lim_{x \to -1^+} f(x) = 1$. True

12. Find $\lim_{x \to 2} f(x) = DNE$. False

13. Find $\lim_{x \to 2} f(x) = 2$. False



14. f(2) = 2 True

15. Find $\lim_{x \to 1^-} f(x) = 2$. True

16. Find $\lim_{x \to 1^+} f(x) = 1$. True

17. Find $\lim_{x \to 1} f(x) = DNE$. True

18. Find $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$. True

19. $\lim_{x \to c} f(x)$ exists at every c in (-1,1) True
[note: () means that the endpoints are not included].

20. $\lim_{x \to c} f(x)$ exists at every c in (1,3). True
[note: () means that the endpoints are not included].

21. Given the function, $f(x) = 2x^2 - 5x$

a. Find the average rate of change over x-interval [0,3].

b. Find the expression for the instantaneous rate of change at any x value.

a) $f(0) = 2(0)^2 - 5(0)$

$= 0$

$f(3) = 2(3)^2 - 5(3)$

$18 - 15$

$f(3) = 3$

$\frac{3-0}{3-0} = \boxed{1}$

b) $\lim_{h \to 0} \frac{2(x+h)^2 - 5(x+h) - (2x^2 - 5x)}{h}$

$= \frac{2(x^2 + 2xh + h^2) - 5x - 5h - 2x^2 + 5x}{h}$

$= \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-5x} - 5h \cancel{-2x^2} \cancel{+5x}}{h}$

$= \frac{4xh + 2h^2 - 5h}{h} = 4x + 2h - 5$

$= \boxed{4x - 5}$

22. Find f'(x) if $f(x) = \sqrt{x+2}$.

$\lim_{h \to 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \cdot \frac{\sqrt{x+h+2} + \sqrt{x+2}}{\sqrt{x+h+2} + \sqrt{x+2}} = \frac{x+h+2 - (x+2)}{h\sqrt{x+h+2} + h\sqrt{x+2}} =$

$\frac{x+h+2-x-2}{h\sqrt{x+h+2} + h\sqrt{x+2}} = \frac{h}{h\sqrt{x+h+2} + h\sqrt{x+2}} = \frac{1}{\sqrt{x+h+2} + \sqrt{x+}}$

$= \frac{1}{\sqrt{x+2} + \sqrt{x+2}} = \frac{1}{2\sqrt{x+2}}$

$\boxed{f'(x) = \frac{1}{2\sqrt{x+2}}}$